1- (MAT-W 2016-HL-Paper 2-Q10)-*FUNCTIONS - ROOTS, DIFFERENTIATION, INTEGRATION*

Let the function f be defined by $f(x) = \dfrac{2-\mathrm{e}^x}{2\mathrm{e}^x - 1}$, $x \in D$.

(a) Determine D, the largest possible domain of f. [2]

(b) Show that the graph of f has three asymptotes and state their equations. [5]

(c) Show that $f'(x) = -\dfrac{3\mathrm{e}^x}{\left(2\mathrm{e}^x - 1\right)^2}$. [3]

(d) Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. [4]

(e) Find an expression for $f^{-1}(x)$. [4]

(f) Consider the region R enclosed by the graph of $y = f(x)$ and the axes.
Find the volume of the solid obtained when R is rotated through 2π about the y-axis. [4]

2- (MAT-W 2016-HL-Paper 2-Q5)-*FUNCTIONS - ROOTS, GRAPHS*

Consider the function f defined by $f(x) = 3x \arccos(x)$ where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points. [3]

(b) State the range of f. [2]

(c) Solve the inequality $|3x \arccos(x)| > 1$. [4]

3- (MAT-S 2016-HL-Paper 2/1-Q11)-*FUNCTIONS - ROOTS*

Let $f(x) = x^4 + 0.2x^3 - 5.8x^2 - x + 4$, $x \in \mathbb{R}$.

(a) Find the solutions of $f(x) > 0$. [3]

(b) For the curve $y = f(x)$.

(i) Find the coordinates of both local minimum points.

(ii) Find the x-coordinates of the points of inflexion. [5]

The domain of f is now restricted to $[0, a]$.

(c) (i) Write down the largest value of a for which f has an inverse. Give your answer correct to 3 significant figures.

(ii) For this value of a sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing clearly the coordinates of the end points of each curve.

(iii) Solve $f^{-1}(x) = 1$. [6]

Let $g(x) = 2\sin(x-1) - 3$, $-\frac{\pi}{2} + 1 \le x \le \frac{\pi}{2} + 1$.

(d) (i) Find an expression for $g^{-1}(x)$, stating the domain.

(ii) Solve $\left(f^{-1} \circ g\right)(x) < 1$. [8]

4- (MAT-S 2016-HL-Paper 2/2-Q5)-*FUNCTIONS - ROOTS*

[Maximum mark: 6]

The function f is defined as $f(x) = \sqrt{\frac{1-x}{1+x}}, -1 < x \leq 1$.

Find the inverse function, f^{-1} stating its domain and range.

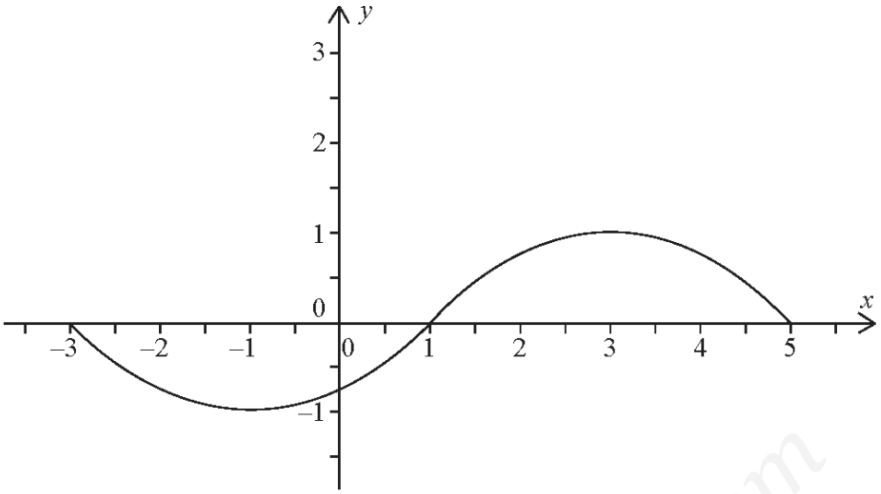
5- (MAT-S 2016-HL-Paper 2/1-Q2)-*FUNCTIONS - ROOTS, QUADRATICS*

(a) Express x^2+4x-2 in the form $(x+a)^2+b$ where $a, b \in \mathbb{Z}$. [2]

(b) If $f(x)=x+2$ and $(g \circ f)(x)=x^2+4x-2$ write down $g(x)$. [2]

6- (MAT-W 2015-HL-Paper 2-Q12)-*FUNCTIONS - ROOTS*

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.
The function has a maximum at $(3, 1)$ and a minimum at $(-1, -1)$.



(a) The functions u and v are defined as $u(x) = x - 3$, $v(x) = 2x$ where $x \in \mathbb{R}$.

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$. [7]

(b) (i) Explain why f does not have an inverse.

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}. State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints. [6]

Consider the function defined by $h(x) = \dfrac{2x-5}{x+d}$, $x \neq -d$ and $d \in \mathbb{R}$.

(c) (i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \dfrac{2x}{x+1}$, $x \neq -1$.

(iii) Find $k(x)$. [8]

7- (MAT-S 2015-HL-Paper 2/1-Q10)-*FUNCTIONS - ROOTS*

A function f is defined by $f(x) = (x+1)(x-1)(x-5)$, $x \in \mathbb{R}$.

(a) Find the values of x for which $f(x) < |f(x)|$. [3]

A function g is defined by $g(x) = x^2 + x - 6$, $x \in \mathbb{R}$.

(b) Find the values of x for which $g(x) < \frac{1}{g(x)}$. [7]

8- (MAT-S 2015-HL-Paper 2/2-Q3)-*FUNCTIONS - ROOTS, GRAPHS*

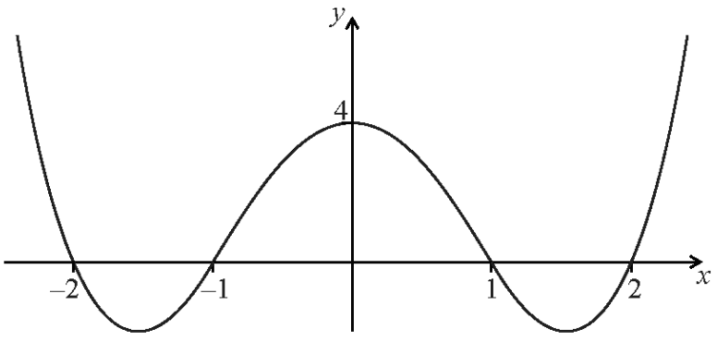
(a) Sketch the graph of $y=(x-5)^2-2|x-5|-9$, for $0 \le x \le 10$. [3]

(b) Hence, or otherwise, solve the equation $(x-5)^2-2|x-5|-9=0$. [2]

9- (MAT-S 2014-HL-Paper 2/1-Q12)-*FUNCTIONS - ROOTS, GRAPHS*

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.



(i) Find the value of $(f \circ g)(1)$.

(ii) Find the value of $(f \circ g \circ g)(1)$.

(iii) Sketch the graph of $y = (f \circ g)(x)$. *[5]*

(b) (i) Sketch the graph of $y = f(x)$.

(ii) State the zeros of f. *[3]*

(c) (i) Sketch the graph of $y = (f \circ f)(x)$.

(ii) State the zeros of $f \circ f$. *[3]*

9- (MAT-S 2014-HL-Paper 2/1-Q12)-*FUNCTIONS - ROOTS, GRAPHS*

(d) Given that we can denote $\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}}$ as f^n,

(i) find the zeros of f^3;

(ii) find the zeros of f^4;

(iii) deduce the zeros of f^8. *[3]*

(e) The zeros of f^{2n} are $a_1, a_2, a_3, \ldots, a_N$.

(i) State the relation between n and N;

(ii) Find, and simplify, an expression for $\sum_{r=1}^{N} |a_r|$ in terms of n. *[4]*

10- (MAT-S 2014-HL-Paper 2/1-Q10)-*FUNCTIONS - ROOTS, DIFFERENTIATION*

Let $f(x) = \dfrac{e^{2x}+1}{e^x - 2}$.

(a) Find the equations of the horizontal and vertical asymptotes of the curve $y = f(x)$. *[4]*

(b) (i) Find $f'(x)$.

(ii) Show that the curve has exactly one point where its tangent is horizontal.

(iii) Find the coordinates of this point. *[8]*

(c) Find the equation of L_1, the normal to the curve at the point where it crosses the y-axis. *[4]*

The line L_2 is parallel to L_1 and tangent to the curve $y = f(x)$.

(d) Find the equation of the line L_2. *[5]*

11- (MAT-S 2014-HL-Paper 2/2-Q7)-*FUNCTIONS - ROOTS, GRAPHS*

The function f is defined as $f(x) = -3 + \frac{1}{x-2}$, $x \neq 2$.

(a) (i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.

(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts. *[4]*

(b) Find the inverse function f^{-1}, stating its domain. *[4]*

12- (MAT-W 2013-HL-Paper 2-Q13)-*FUNCTIONS - ROOTS, EXPONENTIALS - LOGARITHMS, INTEGRATION*

A function f is defined by $f(x)=\frac{1}{2}\left(e^{x}+e^{-x}\right)$, $x\in\mathbb{R}$.

(a) (i) Explain why the inverse function f^{-1} does not exist.

(ii) Show that the equation of the normal to the curve at the point P where $x=\ln 3$ is given by $9x+12y-9\ln 3-20=0$.

(iii) Find the x-coordinates of the points Q and R on the curve such that the tangents at Q and R pass through $(0, 0)$. *[14]*

(b) The domain of f is now restricted to $x\geq 0$.

(i) Find an expression for $f^{-1}(x)$.

(ii) Find the volume generated when the region bounded by the curve $y=f(x)$ and the lines $x=0$ and $y=5$ is rotated through an angle of 2π radians about the y-axis. *[8]*

13- (MAT-S 2012-HL-Paper 2/1-Q6)-*FUNCTIONS - ROOTS*

[Maximum mark: 5]

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

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